

A COMPUTER PROGRAM  
FOR VLF RAY TRACING  
IN A MODEL IONOSPHERE\*

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May 1967

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\*Research supported in part by the Office of Naval  
Research under contract Nonr 1509 (06), and by the  
National Aeronautics and Space Administration under  
contract NGR-16-001-043.

NGR

## ABSTRACT

The ray tracing equations and the expressions necessary to evaluate these equations for a model ionosphere with a centered dipole magnetic field and an  $H^+$ ,  $He^+$ ,  $O^+$ ,  $e^-$  diffusive equilibrium concentration model are given and discussed.

A basic computer program for the numerical calculation of ray paths in a meridian plane is listed and explained. An additional routine to calculate point-to-point ray paths is briefly described. Examples are given of the computer printout for this ray tracing program and of a plotted ray path.

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## 1.0 INTRODUCTION

Since the study of very-low-frequency (VLF) whistlers first began in 1894 [Preece, 1894], many experimental and theoretical investigations have been undertaken to characterize and to explain the properties of naturally occurring VLF radio noises. Helliwell [1965] gives an excellent review of the history, the current experimental characteristics as determined by ground-based and satellite-borne receivers, and the present theoretical interpretations of whistlers, VLF emissions and related ionospheric radio noise phenomena.

Recently satellite observations of signals from VLF ground station transmitters also have been reported by Leiphart [1962], Heyborne [1966], Aubry [1967] and Storey [1967].

A powerful and important analytical method for studying the propagation characteristics of whistlers, VLF emissions and VLF ground station transmissions in the ionosphere is that of ray tracing. In a medium such as the ionosphere and magnetosphere, the trajectory of energy flow for an electromagnetic wave is the ray path for this wave. Therefore, with the ray path method, the trajectory of energy from a lightning discharge or VLF ground station at the base of the ionosphere or from a plasma instability in the ionosphere itself to an observation point can be determined. This ray path

method can be used to produce new explanations for the observed characteristics of whistlers, VLF emissions and ground station transmissions to test existing explanations, or to predict new ionospheric noise phenomena.

The first calculation of whistler ray paths in a model ionosphere was performed by Maeda and Kimura [1956]. They developed a ray theory based on Fermat's principle and used the quasi-longitudinal approximation for the refractive index as given by Storey [1953]. Maeda and Kimura calculated whistler ray paths and whistler dispersions were graphically determined (assuming only electrons in the ionosphere) for different latitudes. The results of these calculations were compared to whistler observations at high and low latitudes with satisfactory qualitative agreement.

Haselgrove [1955] and Haselgrove and Haselgrove [1960] derived a set of first order differential equations appropriate for numerical integration on high speed digital computers. Haselgrove [1957] presented calculations of whistler ray paths using his two-dimensional cartesian ray tracing equations, the quasi-longitudinal approximation to the whistler index of refraction, and the assumption of electrons only and of horizontal stratification. The resulting ray paths were in reasonable quantitative agreement with those of Maeda and Kimura [1956].

Yabroff [1961] published the first whistler ray path calculations which used the exact expression for the refractive index and which included the curvature of the ionosphere, although he assumed an ionosphere composed of electrons only. From his calculations of ray paths for different frequencies, different latitudes, and different initial wave normal angles, he graphically summarized conclusions about the behavior of ray paths in the ionosphere.

When the effects of ions are included in the index of refraction and ray tracing equations, Hines [1957] has shown that for VLF it is possible to have transverse propagation and to have rays refracted back toward the earth at low altitudes. Hines, Hoffman, and Weil [1959] did ray tracing including protons for the special case of transpolar propagation. Kimura [1966] published the first ray tracing results which included effects of the three dominant ions in the upper ionosphere  $H^+$ ,  $He^+$ , and  $O^+$ . He presents ray paths resulting from the inclusion of no ions, one ion, and three ions. The main purpose of his ray tracing study was to confirm the interpretation of the subprotonospheric whistler given by Smith [1964]. The ray tracing results agreed quantitatively with the observational data on subprotonospheric whistlers. This work of Kimura represents the first attempt to completely describe an ionospheric radio noise phenomena in terms of ray tracing.

Edgar and Smith [1966] have used a computer ray tracing program including the effects of ions to understand a type of whistler observed near the geomagnetic equator with OGO-I. By ray tracing they were able to predict the dispersion, shape, spacing in time, and nose frequencies of these anomalous nose whistlers.

Shawhan [1967] has used the computer ray tracing program discussed in this paper to calculate ray paths in a model ionosphere with ions for variations in frequency, initial latitude, initial altitude, initial wave normal angle, and model ionosphere. Frequency-time spectrograms for both positive and negative ion cyclotron whistlers have been computed, and explanations of the hook, riser, and check whistlers are proposed based on ray tracing results.

This report discusses a computer ray tracing program for the numerical computation of ray paths in a model ionosphere including heavy ions and a dipole magnetic field. This basic program can be adapted to problems of explaining new whistlers observed in satellites, of studying wave normal behavior of whistlers and VLF ground transmitter signals, of defining the conditions for ducting of whistlers along magnetic field lines, of explaining propagation effects of VLF radio noise such as the low frequency cutoff of VLF hiss and chorus noise bands, and of calculating ray behavior in the outer magnetosphere, the solar wind and in other planetary ionospheres such as for Jupiter.

In Section 2 the conditions for the validity of ray tracing results are discussed and the Haselgrove [1955] three-dimensional ray tracing equations are given and discussed. These equations are specialized to ray tracing in a magnetic meridian and the expressions needed to evaluate these equations for a model ionosphere are given in the notation of Stix [1962].

A flow chart of a computer program to calculate ray paths in a model ionosphere is described in Section 3. In Section 4 a Fortran IV computer program is listed and its performance discussed. Examples are given of the resulting computer printout and of a sample ray path.

Comments are made in Section 5 on how the basic program of Section 4 can be adapted for special ray tracing problems. A routine is discussed for finding the ray paths between two points in the model ionosphere.

## 2.0 RAY TRACING EQUATIONS

### 2.1 Conditions for Validity of Ray Tracing Equations.

Two conditions must be satisfied in order to validly use the ray tracing approach to energy propagation in a model ionosphere. Because the expressions for the index of refraction given in Section 2.3 are derived with the assumption of plane waves [see Stix, 1962], the condition for the WKB approximation must be satisfied. In terms of the index of refraction this condition can be written

$$\left| \frac{1}{n^2} \frac{dn}{ds} \frac{c}{\omega} \right| \ll 1, \quad (1)$$

where  $n$  is the phase index of refraction along the ray path,  $dn/ds$  is the derivative of the index of refraction along the ray path and  $\omega$  is the wave frequency. Equation (1) is generally satisfied if near a reflection point  $dn/ds$  goes to zero faster than  $n^2$ , if  $dn/ds$  is not large, as it could be for large density or temperature gradients in the ionosphere, or if  $\omega$  is not too low.

Because the Haselgrove [1955] ray tracing equations are based on a Poynting vector approach, the ionosphere must be non-absorbing; the collision frequency (charged-charged and neutral-charged) in the ionosphere must be much less than the wave frequency of interest. If the medium is absorbing

Hines [1951] has shown that a wave packet approach gives the more physical results. For the Earth ionosphere and wave frequencies above 100 Hz, ray tracing is generally valid above approximately 100 km.

## 2.2 Three-Dimensional Haselgrove Equations

Haselgrove [1955] and Haselgrove and Haselgrove [1960] made an important contribution to the problem of ray tracing by deriving a closed set of first order differential equations in general curvilinear coordinates. This general set of equations can be written in spherical polar coordinates and both the centered dipole magnetic field and the concentration gradients can be treated without the assumption of horizontal stratification. Because these differential equations are first order in an independent variable (phase time), they are readily integrated numerically in a digital computer.

The three-dimensional set of ray tracing equations appropriate for tracing ray paths in a model ionosphere is as follows:

$$\frac{dr}{dt} = \frac{1}{n^2} \left( \rho_r - n \frac{\partial n}{\partial \rho_r} \right)$$

$$\frac{d\theta}{dt} = \frac{1}{n^2} \left( \rho_\theta - n \frac{\partial n}{\partial \rho_\theta} \right)$$

$$\frac{d\phi}{dt} = \frac{1}{r n^2 \sin \theta} \left( \rho_\phi - n \frac{\partial n}{\partial \rho_\phi} \right)$$

$$\frac{d\rho_r}{dt} = \frac{1}{n} \frac{\partial n}{\partial r} + \rho_\theta \frac{d\theta}{dt} + \rho_\phi \frac{d\phi}{dt} \sin \theta \quad (2)$$

$$\frac{d\rho_\theta}{dt} = \frac{1}{r} \left( \frac{1}{n} \frac{\partial n}{\partial \theta} - \rho_\theta \frac{dr}{dt} + r \rho_\phi \frac{d\phi}{dt} \cos \theta \right)$$

$$\frac{d\rho_{\varnothing}}{dt} = \frac{1}{r \sin \theta} \left( \frac{1}{n} \frac{\partial n}{\partial \varnothing} - \rho_{\varnothing} \frac{dr}{dt} \sin \theta - r \rho_{\varnothing} \frac{d\theta}{dt} \cos \theta \right) .$$

where  $r$ ,  $\theta$ ,  $\varnothing$  are the spherical polar coordinates,  $n$  is the phase refractive index,  $\rho_r$ ,  $\rho_{\theta}$ ,  $\rho_{\varnothing}$  are the components of the index of refraction  $n^2 = \rho_r^2 + \rho_{\theta}^2 + \rho_{\varnothing}^2$ , and  $t$  is the phase time along the ray path.

One important feature of ray tracing with spherical polar coordinates in the ionosphere not emphasized by Haselgrove [1955] is that there is no generalized form of Snell's law (see Kelso, 1964). Only in the case that  $\nabla n$  is a constant vector for a given  $\vec{n}$ , can a form of Snell's law be derived from this generalized coordinate treatment.

### 2.3 Two Dimensional Equations and Expressions for a Model Ionosphere.

For ray tracing in a model Earth ionosphere the equations (2) can be specialized to expressions for ray tracing in a magnetic meridian. For the case that  $\rho_\theta = 0$  initially and where  $\partial n / \partial \theta = 0$  and  $\partial n / \partial \rho_\theta = 0$ , the ray must stay in the magnetic meridian.

Neglect of deviations of the ray path in the  $\theta$  direction is a good first approximation when a centered dipole approximation for the magnetic field is used, and when the magnitude of concentration gradients in the  $\theta$  direction is considered even for  $\rho_\theta \neq 0$  initially. The expression for a dipole magnetic field has no  $\theta$  dependence, therefore the  $\theta$  component of  $B$  is zero. Also, the longitudinal (diurnal) gradients in the electron and ion concentrations in the ionosphere are generally much smaller than the gradients with radial distance and with latitude and, too, the index of refraction is greater perpendicular to the magnetic meridian so that lightning energy tends to be refracted into the magnetic meridian. Therefore, the refractive index has a small  $\theta$  and  $\rho_\theta$  dependence and the ray path would not be expected to deviate appreciably

in the  $\varnothing$  direction. The two dimensional ray tracing equations to be integrated are

$$\begin{aligned}
 \frac{dr}{dt} &= \frac{1}{n^2} \left( \rho_r - n \frac{\partial n}{\partial \rho_r} \right) \\
 \frac{d\varnothing}{dt} &= \frac{1}{rn^2} \left( \rho_\varnothing - \frac{\partial n}{\partial \rho_\varnothing} \right) \\
 \frac{d\rho_r}{dt} &= \frac{1}{n} \frac{\partial n}{\partial r} + \rho_\varnothing \frac{d\varnothing}{dt} \\
 \frac{d\rho_\varnothing}{dt} &= \frac{1}{r} \left( \frac{1}{n} \frac{\partial n}{\partial \varnothing} - \rho_\varnothing \frac{dr}{dt} \right) .
 \end{aligned} \tag{3}$$

In addition, differential equations for the path length along the ray path and the group delay time are given by Haselgrove [1955]:

$$\frac{dS}{dt} = \frac{1}{n^2} \left( n^2 + \left( \frac{\partial n}{\partial \psi} \right)^2 \right)^{1/2} \tag{4}$$

$$\frac{dT}{dt} = \frac{1}{c} \left( 1 + \frac{\omega}{n} \frac{\partial n}{\partial \omega} \right) \tag{5}$$

where  $\psi$  is the angle between the magnetic field direction and the index of refraction vector  $\vec{n}$ ,  $\omega$  is the wave frequency in  $\text{rad sec}^{-1}$ , and  $c$  is the velocity of light in a vacuum.

In deriving the explicit forms for the quantities in the two dimensional set of ray tracing equations (3), (4) and (5), Yabroff [1961] and Kimura [1966] used the Appleton-Hartree formulation for the index of refraction. We present the expressions for  $n$ ,  $\frac{\partial n}{\partial \rho_r}$ ,  $\frac{\partial n}{\partial \rho_\theta}$ ,  $\frac{\partial n}{\partial r}$ ,  $\frac{\partial n}{\partial \theta}$ , and  $\frac{\partial n}{\partial \omega}$  in the cold plasma formulation of Stix [1962].

### 1. Phase Index of Refraction $n$

According to Stix  $n^2$  is the solution to the quadratic equation

$$An^4 - Bn^2 + C = 0 \quad (6)$$

in the form

$$n^2 = \frac{B + F}{2A} \quad (7)$$

$$\text{where } A = S \sin^2 \psi + P \cos^2 \psi \quad (8)$$

$$B = RL \sin^2 \psi + PS (1 + \cos^2 \psi) \quad (9)$$

$$C = PRL \quad (10)$$

$$F^2 = (RL - PS)^2 \sin^4 \psi + 4 P^2 D^2 \cos^2 \psi \quad (11)$$

and

$$R = 1 - \sum_k \frac{\pi k^2}{\omega} \frac{1}{(\omega + \epsilon_k \Omega_k)} \quad (12)$$

$$L = 1 - \sum_k \frac{\pi_k^2}{\omega} \frac{1}{(\omega - \epsilon_k \Omega_k)} \quad (13)$$

$$P = 1 - \sum_k \frac{\pi_k^2}{\omega^2} \quad (14)$$

$$D = \frac{1}{2} (R - L) \quad (15)$$

$$S = \frac{1}{2} (R + L) \quad (16)$$

In the expressions (12) to (14) we have used

$$\pi_k^2 = \frac{4\pi n_k e^2}{m_k} \quad \text{plasma frequency squared} \quad (17)$$

$$\Omega_k = \left| \frac{e B}{m_k c} \right| \quad \text{gyrofrequency} \quad (18)$$

$$\epsilon_k = e / |e| \quad \text{sign of electronic charge} \quad (19)$$

and the summation index  $k$  represents all ions and the electrons. Therefore  $m_k$  is the mass and  $n_k$  the number density for the  $k^{\text{th}}$  constituent. The sign to be used in equation (7) for the phase refractive index is determined by the particular region of the CMA diagram and by the desired mode [see Stix, 1962]. For the electron whistler mode [ $\omega > \Omega_1$ ] the minus sign gives the correct branch for up going waves. For up going waves in the ion cyclotron mode (proton whistler) the plus signs gives the correct branch. In the case of

downgoing electron whistler waves the sign must be changed from minus to plus for the case that the wave frequency is less than the ion cyclotron frequency (see Shawhan, 1967).

## 2. Derivatives of the form $\partial n / \partial \rho_1$

Derivatives of this form can be derived from the expression

$$\frac{\partial n}{\partial \rho_1} = \frac{\partial n}{\partial \psi} \frac{\partial \psi}{\partial \rho_1} \quad (20)$$

$$(a) \frac{\partial \psi}{\partial \rho_1}$$

Noting again that  $\rho_r$  and  $\rho_\theta$  are the components of the index of refraction in the  $r$ ,  $\theta$  directions respectively,  $\partial n / \partial \rho_1$  is derived from the definition of the angle  $\psi$ :

$$\cos \psi = \frac{\rho_r B_r + \rho_\theta B_\theta}{\rho B}$$

which yields

$$\frac{\partial \psi}{\partial \rho_1} = \frac{B \cos \psi \rho_1 - \rho B_1}{\rho^2 B \sin \psi} \quad (21)$$

$$(b) \frac{\partial n}{\partial \psi}$$

An expression for  $\partial n / \partial \psi$  is obtained by differentiating equation (6) with respect to  $\psi$ :

$$\frac{\partial n}{\partial \psi} = - \frac{n^4 \frac{\partial A}{\partial \psi} - n^2 \frac{\partial B}{\partial \psi} + \frac{\partial C}{\partial \psi}}{4An^3 - 2Bn} \quad (22)$$

From differentiation of (8), (9), and (10) with respect to  $\psi$  we have

$$\frac{\partial A}{\partial \psi} = 2 (S - P) \sin \psi \cos \psi$$

$$\frac{\partial B}{\partial \psi} = 2 (RL - PS) \sin \psi \cos \psi$$

$$\frac{\partial C}{\partial \psi} = 0 . \quad (23)$$

The expressions (23) are used in (22) and the product of (22) with (21) gives  $\partial n / \partial \rho_1$ .

3. Derivatives of the form  $\partial n / \partial x_1$  where  $x_1 = (r, \theta)$

The general expression for  $\partial n / \partial x_1$  is given by

$$\frac{\partial n}{\partial x_1} = \sum_k \frac{\partial n}{\partial \pi_k^2} \frac{\partial \pi_k^2}{\partial x_1} + \sum_k \frac{\partial n}{\partial \Omega_k} \frac{\partial \Omega_k}{\partial x_1} + \frac{\partial n}{\partial \psi} \frac{\partial \psi}{\partial x_1} \quad (24)$$

Each term of (24) must now be evaluated.

$$(a) \quad \frac{\partial n}{\partial \pi_k^2}$$

In the same manner as for (22) we obtain

$$\frac{\partial n}{\partial \pi_k^2} = - \frac{n^4 \frac{\partial A}{\partial \pi_k^2} - n^2 \frac{\partial B}{\partial \pi_k^2} + \frac{\partial C}{\partial \pi_k^2}}{4An^3 - 2Bn} \quad (25)$$

where (8), (9), and (10) are used to obtain the derivatives:

$$\frac{\partial A}{\partial \pi_k^2} = \frac{1}{2} \left( \frac{\partial R}{\partial \pi_k^2} + \frac{\partial L}{\partial \pi_k^2} \right) \sin^2 \psi + \frac{\partial P}{\partial \pi_k^2} \cos^2 \psi$$

$$\begin{aligned} \frac{\partial B}{\partial \pi_k^2} = & \left( R \frac{\partial L}{\partial \pi_k^2} + L \frac{\partial R}{\partial \pi_k^2} \right) \sin^2 \psi + \left( \frac{1}{2} P \left( \frac{\partial R}{\partial \pi_k^2} + \frac{\partial L}{\partial \pi_k^2} \right) \right. \\ & \left. + S \frac{\partial P}{\partial \pi_k^2} \right) (1 + \cos^2 \psi) . \end{aligned}$$

$$\frac{\partial C}{\partial \pi_k^2} = P \left( R \frac{\partial L}{\partial \pi_k^2} + L \frac{\partial R}{\partial \pi_k^2} \right) + RL \frac{\partial P}{\partial \pi_k^2} \quad (26)$$

where the derivatives of R, L, and P are obtained from (12), (13), and (14):

$$\begin{aligned} \frac{\partial R}{\partial \pi_k^2} &= - \frac{1}{\omega(\omega + \epsilon_k \Omega_k)} \\ \frac{\partial L}{\partial \pi_k^2} &= - \frac{1}{\omega(\omega - \epsilon_k \Omega_k)} \\ \frac{\partial P}{\partial \pi_k^2} &= - 1/\omega^2 \end{aligned} \quad (27)$$

Equations (27) are used in equations (26) to give the expression (25).

$$(b) \quad \frac{\partial n}{\partial \Omega_k}$$

As for (22) and (25)

$$\frac{\partial n}{\partial \Omega_k} = - \frac{n^4 \frac{\partial A}{\partial \Omega_k} - n^2 \frac{\partial B}{\partial \Omega_k} + \frac{\partial C}{\partial \Omega_k}}{4An^3 - 2Bn^2} \quad (28)$$

and

$$\begin{aligned}\frac{\partial A}{\partial \Omega_k} &= \frac{1}{2} \left( \frac{\partial R}{\partial \Omega_k} + \frac{\partial L}{\partial \Omega_k} \right) \sin^2 \psi \\ \frac{\partial B}{\partial \Omega_k} &= \left( R \frac{\partial L}{\partial \Omega_k} + L \frac{\partial R}{\partial \Omega_k} \right) \sin^2 \psi + \frac{1}{2} P \left( \frac{\partial R}{\partial \Omega_k} + \frac{\partial L}{\partial \Omega_k} \right) \\ &\quad (1 + \cos^2 \psi) \\ \frac{\partial C}{\partial \Omega_k} &= P \left( L \frac{\partial R}{\partial \Omega_k} + R \frac{\partial L}{\partial \Omega_k} \right) \quad (29)\end{aligned}$$

and for (29)

$$\begin{aligned}\frac{\partial R}{\partial \Omega_k} &= \frac{\pi_k^2 \epsilon_k}{\omega (\omega + \epsilon_k \Omega_k)^2} \\ \frac{\partial L}{\partial \Omega_k} &= \frac{-\pi_k^2 \epsilon_k}{\omega (\omega - \epsilon_k \Omega_k)^2} \\ \frac{\partial P}{\partial \Omega_k} &= 0 \quad (30)\end{aligned}$$

The equations (30) are used in (29) to obtain  $\partial n / \partial \Omega_k$  by (28).

$$(c) \quad \frac{\partial \pi_k^2}{\partial x_1}$$

It is assumed, though not necessarily so, that  $\frac{\partial \pi_k^2}{\partial \theta} = 0$ . With more experimental measurements it may be found that the gradients of concentration with latitude are significant. For this present ray tracing program, they are assumed to be zero. From (17) we have that

$$\frac{\partial \pi_k^2}{\partial r} = \frac{4\pi^2 e}{m_k} \frac{\partial n_k}{\partial r} \quad . \quad (31)$$

Modifying the work of Angerami and Thomas [1964] for a diffusive equilibrium model of the ionosphere, we have

$$\frac{n_e(z)}{n_{eo}} = \left[ \sum_{k=1}^3 \alpha_{ko} e^{-z/H_k} \right]^{1/2} = Q^{1/2} \quad (32)$$

where  $z$  is the geopotential height

$$z = r_o (r - r_o)/r \quad (33)$$

for  $r_o$  being a reference altitude at which the fractional concentrations of the ions with respect to

the electrons is given:

$$\alpha_{ko} = \frac{n_{ko}}{n_{eo}} \quad (34)$$

$H_k$  is the scale height for the  $k^{\text{th}}$  ion

$$H_k = kT/m_k g_0 \quad (35)$$

and  $g_0$  is the acceleration of gravity at  $r_0$ .

From these relations we obtain

$$\begin{aligned} n_e &= n_{eo} Q^{1/2} \\ n_k &= n_{eo} \alpha_{ko} e^{-z/H_k} Q^{-1/2} \end{aligned} \quad (36)$$

Finally

$$\begin{aligned} \frac{\partial \pi_e^2}{\partial r} &= \left( \frac{r_0}{r} \right)^2 \frac{1}{2} \frac{1}{Q} \frac{\partial Q}{\partial z} \pi_e^2 \\ \frac{\partial \pi_k^2}{\partial r} &= - \left( \frac{r_0}{r} \right)^2 \left( \frac{1}{H_k} + \frac{1}{2} \frac{1}{Q} \frac{\partial Q}{\partial z} \right) \pi_k^2 \end{aligned} \quad (37)$$

where

$$\frac{\partial Q}{\partial z} = - \sum_{k=1}^3 \frac{\alpha_{ko}}{H_k} e^{-z/H_k} .$$

$$(d) \quad \frac{\partial \omega_k}{\partial x_1}$$

The magnitude of the centered dipole magnetic field is given by

$$B = 0.312 \left( \frac{r_E}{r} \right)^3 (1 + 3 \cos^2 \theta)^{1/2} \quad (38)$$

where  $r_E = 6370$  km, so that

$$\frac{\partial \omega_k}{\partial r} = - \frac{3}{r} \omega_k$$

$$\frac{\partial \omega_k}{\partial \theta} = - \frac{3 \cos \theta \sin \theta}{(1 + 3 \cos^2 \theta)} \omega_k . \quad (39)$$

$$(e) \quad \frac{\partial \psi}{\partial x_1}$$

From the expression

$$\frac{\partial \psi}{\partial x_1} = \frac{\partial \psi}{\partial \gamma} \frac{\partial \gamma}{\partial x_1} \quad (40)$$

$\partial\psi/\partial x_1$  is obtained, where  $\gamma$  is the angle between the magnetic field direction and radius vector  $r$  and is defined by

$$\tan \gamma = \frac{1}{2} \tan \theta . \quad (41)$$

From (41), therefore, for a centered dipole field

$$\begin{aligned} \frac{\partial \gamma}{\partial \theta} &= \frac{1}{2 \cos^2 \theta + \frac{1}{2} \sin^2 \theta} \\ \frac{\partial \gamma}{\partial r} &= 0 . \end{aligned} \quad (42)$$

Since  $B_r = B \cos \gamma$

$$B_\theta = B \sin \gamma \quad (43)$$

the definition of  $\psi$  is used to obtain

$$\frac{\partial \psi}{\partial \gamma} = \frac{\rho_r B_\theta - \rho_\theta B_r}{\rho B \sin \psi} . \quad (44)$$

Equations (40), (39), (37), (31), (28), (25), and (22) are used to evaluate (24).

4. Derivative  $dS/dt$ 

The expressions for  $n^2$  and  $\partial n/\partial \psi$  to be used in equation (4) are given by (7) and (22).

5. Derivative  $dT/dt$ 

For equation (5) we have

$$\frac{\partial n}{\partial \omega} = - \frac{n^4 \frac{\partial A}{\partial \omega} - n^2 \frac{\partial B}{\partial \omega} + \frac{\partial C}{\partial \omega}}{4An^3 - 2Bn^2} \quad (45)$$

and

$$\begin{aligned} \frac{\partial A}{\partial \omega} &= \frac{1}{2} \left( \frac{\partial R}{\partial \omega} + \frac{\partial L}{\partial \omega} \right) \sin^2 \psi + \frac{\partial P}{\partial \omega} \cos^2 \psi \\ \frac{\partial B}{\partial \omega} &= \left( R \frac{\partial L}{\partial \omega} + L \frac{\partial R}{\partial \omega} \right) \sin^2 \psi + \left( \frac{1}{2} P \left( \frac{\partial R}{\partial \omega} + \frac{\partial L}{\partial \omega} \right) + S \frac{\partial P}{\partial \omega} \right) \\ &\quad (1 + \cos^2 \psi) \end{aligned}$$

$$\frac{\partial C}{\partial \omega} = P \left( R \frac{\partial L}{\partial \omega} + \frac{\partial R}{\partial \omega} \right) + RL \frac{\partial P}{\partial \omega} \quad (46)$$

for which

$$\frac{\partial R}{\partial \omega} = \sum_k \frac{\pi_k^2}{\omega^2} \frac{2\omega + \epsilon_k \Omega_k}{(\omega + \epsilon_k \Omega_k)^2}$$

$$\frac{\partial L}{\partial \omega} = \sum_k \frac{\pi_k^2}{\omega^2} \frac{2\omega - \epsilon_k \Omega_k}{(\omega - \epsilon_k \Omega_k)^2}$$

$$\frac{\partial P}{\partial \omega} = 2 \sum_k \frac{\pi_k^2}{\omega^3} . \quad (47)$$

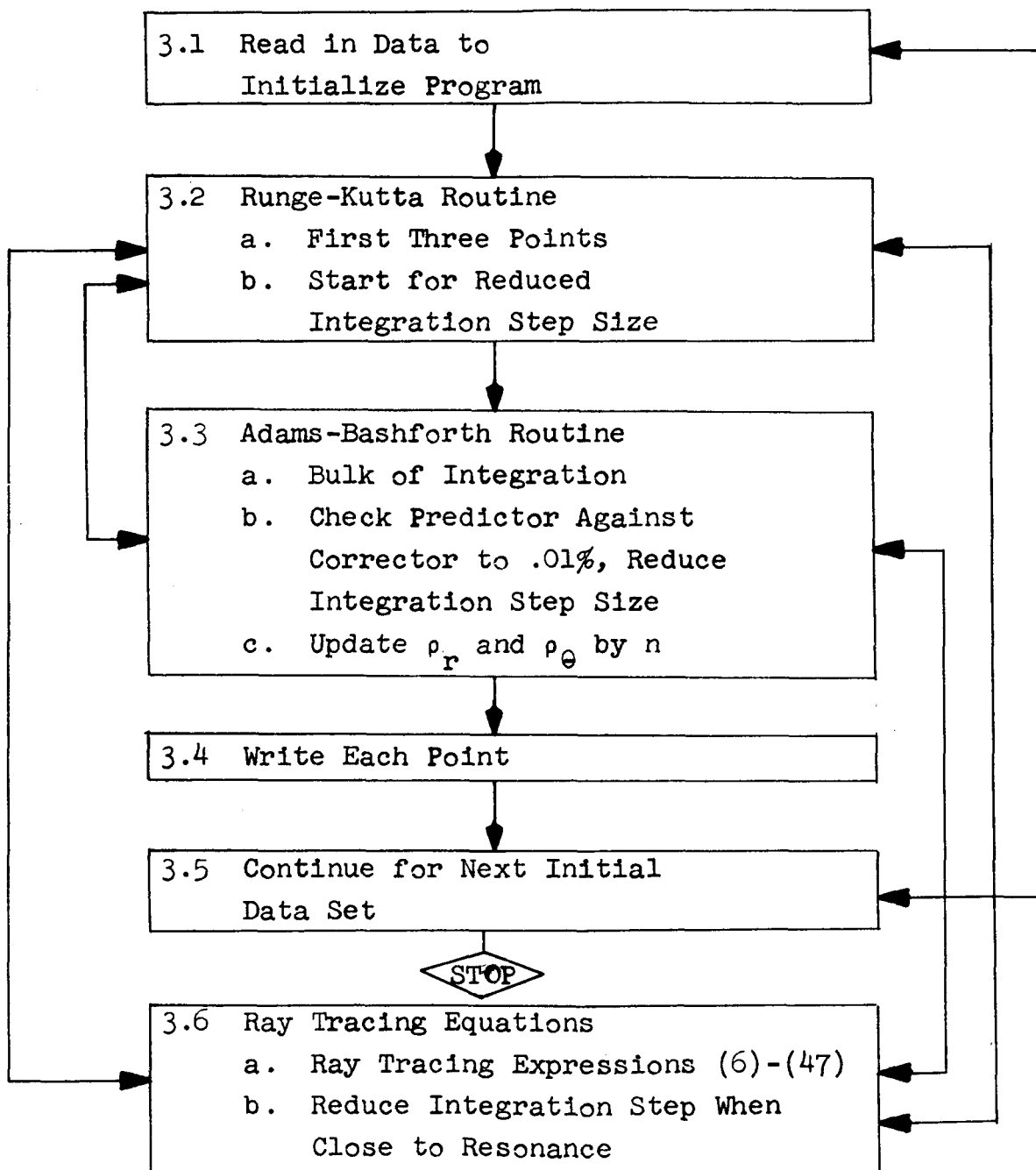
The equations listed in this section are the expressions necessary to calculate the ray path  $(r, \theta)$ , the  $r$  and  $\theta$  components of the index of refraction vector  $n = (\rho_r^2 + \rho_\theta^2)^{1/2}$ , the total path length  $S$ , and the total time delay along the path  $T$ , for a given frequency and a given initial wave normal angle ( $\rho_r/\rho$  and  $\rho_\theta/\rho$  specified). It should be noted that these expressions are equally valid for all frequencies. For HF, however, the equations can be simplified and computation time shortened. For ELF care must be taken in the computer program to avoid addition and subtraction of very large quantities.

### 3.0 FLOW CHART OF COMPUTATIONAL METHOD

The ray tracing equations (3) cannot be solved in general for a model ionosphere with a dipole magnetic field and an exponential-like ionosphere. Ray paths must therefore be calculated numerically as an initial value problem.

In Figure 1 is given a flow chart illustrating a method to efficiently calculate ray paths in a model ionosphere. In Section 3.1 the data to initialize the problem is read in. With these data the Runge-Kutta routine in Section 3.2 is used to calculate the next three points on the ray path. The ray tracing equations are evaluated in Section 3.6 to give the Runge-Kutta predictor and corrector values. These first three points start the Adams-Bashforth routine in Section 3.3 which continues calculating points on the ray path but faster than the Runge-Kutta method. Again the ray tracing equations are evaluated in Section 3.6. The calculated values for each point are then written out (Section 3.4) and a new set of initial data read in (Section 3.5).

Figure 1  
RAY TRACING PROGRAM FLOWCHART



3.1 Read in Data to Initialize Program: Initial values for the ray tracing quantities must be specified. These quantities are  $R_0$  and  $\theta_0$  the polar coordinates of the start of the ray path and  $\Delta_0$  the initial direction of the wave normal angle with respect to the radial direction. Also the wave frequency  $\omega$ , the integer value which specifies the parameters for the model ionosphere  $r_0$ ,  $\alpha_{10}$ ,  $\alpha_{20}$ ,  $T$  and  $n_{e0}$  (see equations (32)-(36) ), the integration step size and the number of points to be calculated.

### 3.2 Runge-Kutta Routine:

a. Once all the starting parameters have been specified the fourth order Runge-Kutta integration routine is used to obtain the next three points along the ray path and the values of the ray tracing differential equations (21) at each point. It is these first four derivatives and values  $r$ ,  $\theta$  (polar coordinates),  $\rho_r$ ,  $\rho_\theta$  ( $r$ -,  $\theta$ -component of index of refraction),  $S$  (path length),  $T$  (group time delay) that are needed to start the fourth order Adams-Bashforth integration routine. Values of the derivatives for the current values of  $r$ ,  $\theta$ ,  $\rho_r$ ,  $\rho_\theta$ ,  $S$ , and  $T$  are obtained by transferring to Part 3.6, Ray Tracing Expressions. For the Runge-Kutta routine three predictors and one corrector for the next point must be evaluated [Abramowitz and Stegun, 1964]. For the Adams-Bashforth routine, however, only one predictor and one corrector is calculated so that this latter routine is faster and therefore used for the bulk of the integration.

b. This Runge-Kutta Routine is also used to obtain the first three points of the ray path after the

integration step is reduced in the Adams-Bashforth routine or in the calculation of the ray tracing expressions. With the reduction of the integration step size the three new points and derivatives of the ray tracing quantities at these points are required to continue in the Adams-Bashforth routine.

### 3.3 Adams-Bashforth Routine

a. Equations for the fourth order Adams-Bashforth integration routine are given by Abranowitz and Stegen [1964]. This routine can be used once the first four starting points have been obtained with the Runge-Kutta routine. Only one predictor and one corrector are required for this routine therefore the time consuming calculation of the ray tracing expressions has to be done only twice for each point. This routine is therefore used for the bulk of the ray path determination.

b. To insure that quantities in Part 3.6 are not changing too fast between integration steps, the predictor is compared to the corrector. If the percentage difference is greater than 0.01%, then the integration interval is halved. The Runge-Kutta routine is then used to start the calculations for the next points with the reduced integration step. This test to 0.01% is made for all of the ray tracing quantities.

c. In Part 3.6 the values of  $\rho_r$  and  $\rho_\theta$  are corrected in the manner suggested by Yabroff [1961]

in order that  $\rho_r^2 + \rho_\theta^2 = n^2$ . In Part 3.3 the values of the predictor and corrector are also corrected for  $\rho_r$  and  $\rho_\theta$  so that no integration error can accumulate. This correction routine has been added to the standard Adams-Bashforth method and has yielded improvement in the integration accuracy.

3.4 Write Each Point: As each point is calculated along the ray path, the values for  $r$  and  $\theta$  (polar coordinates of ray path),  $\Delta$  (wave normal angle),  $n$  (index of refraction),  $S$  (total path length) and  $t$  (time delay along the path) are printed out. Also other quantities of interest can be listed.

3.5 Continue for Next Initial Data Set: With a loop in the program many sets of initial conditions can be read in and the resulting ray paths computed. A series of initial conditions may be desired to study the behavior of ray paths with respect to variation of altitude, latitude, frequency, wave normal angle, or model.

### 3.6 Ray Tracing Equations:

a. It is this Part 3.6 in which the expressions for the gyrofrequencies, plasma frequencies, index of refraction, and all of the necessary derivatives given in Equations (6) to (47) are calculated. From the current values of  $r$ ,  $\theta$ ,  $\rho_r$ ,  $\rho_\theta$ ,  $S$ ,  $T$ , the derivatives given by the ray tracing equations (3), (4), and (5) are evaluated for use in obtaining the next point. The correction of  $\rho_r$  and  $\rho_\theta$  with respect to the calculated value of the index of refraction is made in this part as suggested by Haselgrove [1955] and Yabroff [1961].

b. At a resonance in the index of refraction the index of refraction is large positive on one side of the resonance and large imaginary on the other-side. For a finite integration step it is possible to 'step over' a resonance and to introduce extraneous values of the ray tracing quantities and their derivatives because of this discontinuity. The index of refraction squared is therefore tested at each step. When it becomes negative the integration step size is reduced and the Runge-Kutta routine used until a point

is obtained for which the index of refraction is positive.

Figure 1 gives the scheme of calculation of ray paths in a model ionosphere independent of programming language or specific computer. In the next section a specific ray tracing program is given and discussed.

#### 4.0 FORTRAN IV COMPUTER PROGRAM

Digital computers have made it possible to compute ray paths for a model ionosphere with high speed and accuracy. In Section 4.1 an example of a ray tracing program is given. This program is written in FORTRAN IV and is intended to run on an IBM 7044 computer. This basic program is designed to calculate two-dimensional ray paths in a magnetic meridian for any frequency. The particular model in this program is a diffusive equilibrium model including three ions  $H^+$ ,  $He^+$  and  $O^+$ .

In Section 4.2 this program is described in detail. The performance is discussed in Section 4.3 and in Section 4.4 an example of the computer print-out is illustrated along with the plot of a ray path.

## 4.1 Ray Tracing Program

```

C THIS PROGRAM CALCULATES TWO DIMENSIONAL RAY PATHS FOR ANY
C FREQUENCY IN A MODEL IONOSPHERE WITH A DIPOLE MAGNETIC FIELD
C AND A DIFFUSIVE EQUILIBRIUM COMPOSITION OF H+, HE+, O+
C
C VARIATION WITH FREQUENCY AT 30DEG
C
C Y(1)= RADIAL DISTANCE FROM CENTER OF EARTH IN KM
C Y(2)= COLATITUDE IN RADIANS
C Y(3)= RADIAL COMPONENT OF INDEX OF REFRACTION
C Y(4)= LATITUDE COMPONENT OF INDEX OF REFRACTION
C Y(5)= TOTAL PATH LENGTH IN KM
C Y(6)= TOTAL PATH DELAY TIME IN SECONDS
C
C DIMENSION FREQ1(25),DEL2(25),ALFO(3),Y(6),X(6,4),D2(6,4),D1(6),
1 PPP(6),QQQ(6),VVV(6),SSS(6),P1(6),C1(6),OM(4),DOMDR(4),DOMDTH(4),
2 H(3),ALFR(3),PLFREQ(4),E(4),CONP(4),CONM(4),RR(4),ALL(4), ANR(3),
3 PP(4),DRDPL(4),DLDPL(4),DPDPL(4),DADPL(4),DBDPL(4),DCDPL(4),
4 DNDPL(4),DRDOM(4),DLDOM(4),DADOM(4),DBDOM(4),DCDOM(4),DNDOM(4),
5 DPLDR(4),G5(4),G6(4),G7(4),G8(4),G9(4)
6,CHECK(6)
C EQUIVALENCE (R,Y(1)),(TH,Y(2)),(RHOR,Y(3)),(RHOTH,Y(4)),
C(S,Y(5)),(T,Y(6))
C CALL TRAPS (-1,-1)
C READ(5,1) M1
1 FORMAT(I5)
C DO 1000 M2=1,M1
C READ(5,2) M3,M4,M5,MODEL,R1,THH,DELTA
2 FORMAT(4I5,3F10.4)
C READ(5,3) (FREQ1(J),J=1,M4)
3 FORMAT (7F10.4)
C READ (5,4) (DEL2(J),J=1,M5)
4 FORMAT (7F10.4)
C IF(MODEL.EQ.1)GO TO 5
C GO TO 6
C MID-LATITUDE MODEL MODEL=1
5 RO= 6770.
C ANEO=2.E+5
C ALFO(1)=0.0025
C ALFO(2)=0.0225
C TEMP=800.
C GO TO 10
6 IF (MODEL.EQ.2)GO TO 7
C GO TO 10
C EQUATORIAL MODEL MODEL=2
7 RO=7300.
C ANEO=2.8E+4
C ALFO(1)=0.5
C ALFO(2)=0.0
C TEMP=1500.
10 CONTINUE
C GO=3.98022E+10*(RO**(-2))
C H(1)=8.254016E+02*TEMP/GO
C H(2)=0.25*H(1)

```

```

H(3)=0.0625*H(1)
ALFO(3)=1.-ALFO(2)-ALFO(1)
Z1=R0-6370.
DO 1000 J2=1,M4
FREQ=FREQ1(J2)*6.28318
DO 1000 J1=1,M5
DEL1=DEL2(J1)*0.0174532
DELT=DELTA/SQRT(FREQ*1.59155E-4)
WRITE(6,11)M2,R1,THH,DEL2(J1),FREQ1(J2),M3,DELTA,Z1,ANEO,
1ALFO(1),ALFO(2),TEMP
11 FORMAT(1H1,5X,10HDATA SET =,15/6X,3HR1=,F9.2,2HKM,5X,4HTH1=,F8.2,
13HDEG,5X,5HDEL1=,F8.2,3HDEG,5X,5HFREQ=,F8.0,3HCPS,5X,2HM=,15,5X,
26HDELTA=,F10.2/5X,3HRO=,F8.1,2HKM,5X,4HNEO=,1PE10.3,4HCM-3,5X,
38HALFO(1)=,0PF10.5,5X,8HALFO(2)=,F10.5,5X,5HTEMP=,F6.1,5HDEG K)
WRITE(6,12)
12 FORMAT(1H0,5X,1HR,11X,2HTH,8X,3HDEL,8X,1HN,10X,1HT,9X,3HPSI,9X,
12HNE,9X,2HFE,8X,4HFLHR,7X,4HDELT,8X,2HKK)
N=0
MARK=0
Y(1) =R1 + 6370.
Y(2) = (90.-THH)*0.0174532
Y(5) =0.
Y(6) =0.
C
C BEGIN 4TH ORDER RUNGE-KUTTA INTEGRATION FOR 1ST FOUR POINTS
C
23 DO 25 K=1,4
KS=K
GO TO 100
13 CONTINUE
K=KS
DO 14 L=1,6
X(L,K)=Y(L)
14 D2(L,K)=D1(L)
C IF K=4 TRANSFER TO ADAMS-BASHFORTH INTEGRATION
IF(K.NE.4) GO TO 24
IF(MARK.NE.1) GO TO 25
KK=KP
GO TO 27
24 DO 15 L=1,6
PPP(L)=DELT*D1(L)
15 Y(L)=X(L,K)+0.5*PPP(L)
KS=K
GO TO 100
16 CONTINUE
K=KS
DO 17 L=1,6
QQQ(L)=DELT*D1(L)
17 Y(L)=X(L,K)+0.5*QQQ(L)
KS=K
GO TO 100
18 CONTINUE
K=KS
DO 19 L=1,6
VVV(L)=DELT*D1(L)

```

```

19 Y(L)=X(L,K)+VVV(L)
   KS=K
   GO TO 100
20 CONTINUE
   K=KS
   DO 21 L=1,6
     SSS(L)=DELT*D1(L)
21 X(L,K+1)=X(L,K)+(PPP(L)+2.*QQQ(L)+2.*VVV(L)+SSS(L))*0.16666667
   IF(MARK.EQ.1) GO TO 25
   N=0
   R2=X(1,K+1)-6370.
   TH2=90.-57.2958*X(2,K+1)
   DEL3=57.2958*DEL
   WRITE(6,22)R2,TH2,DEL3,AN1,X(6,K+1),PPSI,ANER,OOM4,ALHR,DELT,K
22 FORMAT(1H,6F11.5,4F11.2,I5)
25 CONTINUE

C
C   BEGIN 4TH ORDER ADAMS-BASHFORTH INTEGRATION FOR REMAINDER OF M3
C
   KP=0
   MM=M3-1
26 DO 50 KK=4,MM
   LLL=0
   JK=0
   DELT=DELTA/SQRT(FREQ*1.59155E-4)
27 DO 28 L=1,6
   Y(L)=X(L,4)+(DELT*(55.*D2(L,4)-59.*D2(L,3)+37.*D2(L,2)
C-9.*D2(L,1))*0.041666667)
28 P1(L)=Y(L)
   MARK=0
   N=4
   KT=KK
   GO TO 100
29 CONTINUE
   KK=KT
C   UPDATE FOR VALUES OF RHOR AND RHOTH
   P1(3)=RHOR
   P1(4)=RHOTH
   DO 30 L=1,6
   Y(L)=X(L,4)+(DELT*(D2(L,2)-5.*D2(L,3)+19.*D2(L,4)+9.*D1(L))
C*.041666667)
30 C1(L)=Y(L)
   DO 38 L=1,6
   CHECK(L)=DIM(P1(L),C1(L))/P1(L)
38 IF(CHECK(L).LT.1.E-04) GO TO 39
   KP=KT
   MARK=1
   N=0
   DELT=0.50*DELT
   DO 40 L=1,6
40 Y(L)=X(L,4)
   GO TO 23
39 JK=1
   KT=KK
   GO TO 100

```

```

31 CONTINUE
   KK=KT
C   UPDATE FOR VALUES OF RHOR AND RHOTH
   C1(3)=RHOR
   C1(4)=RHOTH
   JK=0
   DO 32 L=1,6
32  Y(L)=(251.*C1(L)+19.*P1(L))*0.0037037037
   KT=KK
   GO TO 100
33 CONTINUE
   KK=KT
   R2=Y(1)-6370.
   TH2=90.-57.2958*Y(2)
   DEL3=57.2958*DEL
   WRITE(6,34)R2,TH2,DEL3,AN1,Y(6),PPSI,ANER,OOM4,ALHR,DEL T, KK
34  FORMAT(1H ,6F11.5,4F11.2,15)
   IF(R2.GE.300.)GO TO 36
   WRITE(6,35)
35  FORMAT(1H0,21H RAY HAS REACHED 300KM)
   GO TO 1000
C   STORE PREVIOUS VALUE AND DERIVATIVES
36  DO 37 L=1,6
   X(L,4)=Y(L)
   D2(L,1)=D2(L,2)
   D2(L,2)=D2(L,3)
   D2(L,3)=D2(L,4)
37  D2(L,4)=D1(L)
50 CONTINUE
   GO TO 1000
C
C   RAY TRACING EQUATIONS
C
100 N=N+1
   COSTH=COS(TH)
   SINTH=SIN(TH)
   COSTH2=COSTH**2
   SINTH2=SINTH**2
C
C   CALCULATE GYROFREQUENCIES
C
   COSB=SQRT(1.+3.*COSTH2)
   B0=8.0682E+10*COSB*(R **(-3))
   OM(1)=9.579267E+03*B0
   OM(2)=0.25*OM(1)
   OM(3)=0.0625*OM(1)
   OM(4)=+1836.1388*OM(1)
   OOM1=OM(1)/(2.*3.1416)
   OOM4=OM(4)/(2.*3.1416)
C
C   CALCULATE DERIVATIVES OF GYROFREQUENCIES WRT R, TH
C
   DO 101 I=1,4
   DOMDR(I)=-3.*OM(I)/R
101  DOMDTH(I)=-3.*SINTH*COSTH*OM(I)/(COSB**2)

```

```

C
C   CALCULATE PLASMA FREQUENCIES FROM DIFFUSIVE EQUILIBRIUM MODEL
C
      Z=RO*(R-RO)/R
      Q=ALFO(1)*EXP((-Z)/H(1))+ALFO(2)*EXP((-Z)/H(2))
      C+ALFO(3)*EXP((-Z)/H(3))
C   EXPONENTIAL MODEL ELECTRON DENSITY
      ANER=ANEO*SQRT(Q)
      DO 102 I=1,3
      ALFR(I)=ALFO(I)*(Q**(-1.))*EXP((-Z)/H(I))
102  ANR(I)=ALFR(I)*ANER
      PLFREQ(1)=1.733265E+06*ANR(1)
      PLFREQ(2)=0.433316E+06*ANR(2)
      PLFREQ(3)=0.108329E+06*ANR(3)
      PLFREQ(4)=1.733265E+06*1836.1388*ANER
      CC5=(ALFR(1)+.25*ALFR(2)+.0625*ALFR(3))
      CC6=1.+((OM(4)**2)/PLFREQ(4))
      ALHR=(SQRT((+OM(1))*OM(4)*CC5/CC6))*159155
      TANTH=TAN(TH)
C
C   CALCULATE WAVE NORMAL ANGLE BETWEEN MAGNETIC FIELD AND PROPAGATION
C   VECTOR - ANGLE PSI
C
      BRO=(BO/SQRT(1.+0.25*(TANTH**2)))
      IF(TANTH.LT.0.) BRO=-BRO
      BTHO=0.5*BRO*TANTH
      IF((K.EQ.1).AND.(MARK.NE.1)) GO TO 103
      COSDEL=Y(3)/SQRT(Y(3)**2+Y(4)**2)
      SINDEL=Y(4)/SQRT(Y(3)**2+Y(4)**2)
      GO TO 104
103  COSDEL=COS(DEL1)
      SINDEL=SIN(DEL1)
104  CONTINUE
      DEL=ARCOS(COSDEL)
      IF(SINDEL.LT.0.) DEL=-DEL
      COSPHI=BRO/BO
      SINPHI=BTHO/BO
      COSPSI=COSDEL*COSPHI+SINDEL*SINPHI
      SINPSI=SINDEL*COSPHI-COSDEL*SINPHI
      PSI=ARCOS(COSPSI)
      IF(SINPSI.LT.0.) PSI=-PSI
      PPSI=PSI*57.2958
      COS2=COSPSI**2
      SIN2=SINPSI**2
      FREQ2=FREQ**2
C
C   CALC INDEX OF REFRACTION, R-COMPONENT, AND TH-COMPONENT. AN,
C   RHOR, ROTH
C
      E(1)=1.
      E(2)=1.
      E(3)=1.
      E(4)=-1.
      AR=1.0
      AL=1.0

```

```

P=1.0
DO 105 I=1,4
  CONP(I)=FREQ+(E(I)*OM(I))
  CONM(I)=FREQ-(E(I)*OM(I))
  RR(I)=PLFREQ(I)/(FREQ*CONP(I))
  ALL(I)=PLFREQ(I)/(FREQ*CONM(I))
105 PP(I)=PLFREQ(I)/FREQ2
  DO 106 I=1,4
    AR=AR-RR(I)
    AL=AL-ALL(I)
106 P=P-PP(I)
  SS=0.5*(AR+AL)
  D=0.5*(AR-AL)
  A=SS*SIN2+P*COS2
  B=AR*AL*SIN2+P*SS*(1.+COS2)
  C=AR*AL*P
  F=SQRT(((AR*AL-P*SS)**2)*(SIN2**2)+4.*((P*D)**2)*COS2)
  IF(B.LT.0.) AN2=(B-F)/(2.*A)
  AN2=2.*C/(B+F)
  IF(AN2.GT.0.) GO TO 110
C  REDUCE INTEGRATION STEP IF CLOSE TO RESONANCE
  KP=KT
  N=0
  MARK=1
  DELT=0.25*DELT
  IF(DELT.GT.1.E-08) GO TO 108
  WRITE(6,107)
107 FORMAT(1H0,22HTOO CLOSE TO RESONANCE)
  GO TO 1000
108 DO 109 L=1,6
109 Y(L)=X(L,4)
  GO TO 23
110 CONTINUE
  AN4=AN2**2
  AN3=AN2**1.5
  AN1=AN2**0.5
  RHOR=AN1*COSDEL
  RHOTH=AN1*SINDEL
  RHO=SQRT((RHOR**2)+(RHOTH**2))
  RHOR=RHOR*AN1/RHO
  RHOTH=RHOTH*AN1/RHO
  IF(JK.EQ.1) GO TO 125
  ABPSI=ABS(PSI)
C
C  CALC DERIVATIVE OF N WRT RHOR AND RHOTH - DNDROR, DNDROT
C
  CON=(-4.*AN3*A+2.*B*AN1)
  DADPSI=2.*(SS-P)*SINPSI*COSPSI
  DBDPSI=2.*(AR*AL-P*SS)*SINPSI*COSPSI
  DNDPSI=(AN4*DADPSI-AN2*DBDPSI)/CON
  IF(ABPSI.GT.1.E-08) GO TO 111
  DNDROR=0.
  DNDROT=0.
  GO TO 112
111 DNDROR=DNDPSI*((RHOR*BO*COSPSI-AN1*BRO)/(BO*SINPSI*AN2))

```

DNDROT=DNDPSI\*((RHOTH\*BO\*COSPSI-AN1\*BTHO)/(BO\*SINPSI\*AN2))

C  
C CALC DERIVATIVE OF N WRT PLASMA FREQUENCIES - DNDPL(I)  
C

112 DO 113 I=1,4  
 DRDPL(I)=-1./(FREQ\*CONP(I))  
 DLDPL(I)=-1./(FREQ\*CONM(I))  
 DPDPL(I)=-1./FREQ2  
 DADPL(I)=0.5\*(DRDPL(I)+DLDPL(I))\*SIN2+DPDPL(I)\*COS2  
 DBDPL(I)=(AR\*DLDPL(I)+AL\*DRDPL(I))\*SIN2  
 C+(P\*0.5\*(DRDPL(I)+DLDPL(I))+SS\*DPDPL(I))\*(1.+COS2)  
 DCDPL(I)=P\*(AR\*DLDPL(I)+AL\*DRDPL(I))+AR\*AL\*DPDPL(I)  
 113 DNDPL(I)=(DADPL(I)\*AN4-DBDPL(I)\*AN2+DCDPL(I))/CON

C  
C CALC DERIVATIVE OF N WRT GYROFREQUENCIES - DNDOM(I)  
C

DO 114 I=1,4  
 DRDOM(I)=PLFREQ(I)\*E(I)/(FREQ\*(CONP(I)\*\*2))  
 DLDOM(I)=-PLFREQ(I)\*E(I)/(FREQ\*(CONM(I)\*\*2))  
 DADOM(I)=0.5\*(DRDOM(I)+DLDOM(I))\*(SIN2)  
 DBDOM(I)=(AR\*DLDOM(I)+AL\*DRDOM(I))\*(SIN2)  
 C+P\*0.5\*(DRDOM(I)+DLDOM(I))\*(1.+(COS2))  
 DCDOM(I)=P\*(AR\*DLDOM(I)+AL\*DRDOM(I))  
 114 DNDOM(I)=(DADOM(I)\*AN4-DBDOM(I)\*AN2+DCDOM(I))/(2.\*B\*AN1-4.\*A\*AN3)

C  
C CALC DERIVATIVE OF PLASMA FREQUENCIES WRT R - DPLDR(I)  
C

DZDR=(RO/R)\*\*2  
 DQDZ=-(ALFO(1)\*EXP((-Z)/H(1))/H(1))-(ALFO(2)\*EXP((-Z)/H(2))/H(2))  
 C-(ALFO(3)\*EXP((-Z)/H(3))/H(3))  
 DPLDR(4)=(PLFREQ(4))\*DZDR\*0.5\*DQDZ/Q  
 DO 115 I=1,3  
 DPLDR(I)=PLFREQ(I)\*DZDR\*((-1./H(I))-(0.5\*DQDZ/Q))  
 115 CONTINUE

C  
C CALC DERIVATIVE OF N WRT R, TH - DNDR, DNDTH  
C

IF(ABPSI.GT.1.E-08) GO TO 116  
 DPSIDT=1./(2.\*COSTH2+0.5\*SINTH2)  
 GO TO 117  
 116 DPSIDT=(RHOR\*BTHO-RHOTH\*BRO)/(RHO\*BO\*SINPSI\*(2.\*COSTH2+0.5\*  
 CSINTH2))  
 117 DNDR=0.  
 DNDTH=DNDPSI\*DPSIDT  
 DO 118 I=1,4  
 G5(I)=DNDPL(I)\*DPLDR(I)+DNDOM(I)\*DOMDR(I)  
 G6(I)=DNDOM(I)\*DOMDTH(I)  
 DNDR=DNDR+G5(I)  
 118 DNDTH=DNDTH+G6(I)

C  
C CALC DERIVATIVE OF N WRT FREQUENCY - DNDW  
C

DO 119 I=1,4  
 G7(I)=0.5\*(2.\*FREQ+E(I)\*OM(I))\*PLFREQ(I)/((FREQ\*CONP(I))\*\*2)  
 G8(I)=0.5\*(2.\*FREQ-E(I)\*OM(I))\*PLFREQ(I)/((FREQ\*CONM(I))\*\*2)

```
119 G9(I)=2.*(PLFREQ(I))/(FREQ**3)
```

DSDW=0.

DPDW=0.

SG7=0.

 $SG8=0.$ 

DO 120 I=1,4

$$SG7 = G7(1) + SG7$$
$$SG8 = G8(1) + SG8$$
$$DSDW = DSDW + G7(I) + G8(I)$$

```
120  DPDW=DPDW+G9(I)
```

$$DADW = DSDW * SIN2 + DPDW * COS2$$
$$DBDW = (2. * AR * SG8 + 2. * AL * SG7) * SIN2 + (P * DSDW + SS * DPDW) * (1. +$$

C(COS2))

$$DCDW = P * (2. * AR * SG8 + 2. * AL * SG7) + AR * AL * DPDW$$
$$DNDW = (-AN4 * DADW + AN2 * DBDW - DCDW) / (-CON)$$
C  
C  
C

## DERIVATIVES OF INTEGRATED QUANTITIES

$$D1(1) = (RHOR - AN1 * DNDROR) / AN2$$
$$D1(2) = (RHOTH - AN1 * DNDROT) / (AN2 * R)$$
$$D1(3) = (DNDR/AN1) + RHOTH * D1(2)$$
$$D1(4) = ((DNDTH/AN1) - RHOTH * D1(1)) / R$$
$$D1(5) = (\text{SQRT}(\text{AN2} + (\text{DNDPSI} ** 2))) / \text{AN2}$$
$$D1(6) = (1. + (FREQ * DNDW / AN1)) * 3.335634E-06$$

125 GO TO (13,16,18,20,29,31,33),N

1000 CONTINUE

C  
C  
C

THE DATA INPUT CARDS TO INITIALIZE THE PROBLEM ARE AS FOLLOWS

CARD 1..

COL 5 NUMBER OF DATA SETS WITH DIFFERENT INITIAL ALT AND LAT

**CARD 2..**

COL 5 NUMBER OF DATA POINTS TO BE COMPUTED

COL 10 NUMBER OF FREQUENCIES TO BE COMPUTED

COL 15 NUMBER OF INITIAL WAVE NORMAL ANGLES

COL 20 MODEL IONOSPHERE NUMBER

COL 21-30 INITIAL ALTITUDE IN KM

COL 31-40 INITIAL LATITUDE IN DEGREES

COL 41-50 DELTA-INTEGRATION STEP SIZE

CARD 3..

COL 1-10 FREQUENCY 1, COL 11-20 FREQUENCY 2, ETC IN CPS

CARD 4..

COL 1-10 WAVE NORMAL 1, COL 11-20 WAVE NORMAL 2, ETC IN DEG

CARDS 5-7,8-10, ETC JUST LIKE CARDS 2-4

C

CALL EXIT

END

SENTRY SUI

1  
500      4      1      1      300.      30.      10000.  
1000.      200000.      500000.      1000000.

## 4.2 Description of Program

4.2.1 Ray Tracing Quantities. Throughout the ray tracing program, the  $Y(I)$  are the ray tracing quantities being calculated point by point from the ray tracing equations (3), (4) and (5). As defined at the beginning of the program in Section 4.1 the  $Y(I)$  are as follows:

$Y(1)$  = radial distance from center of earth in km,  $r$

$Y(2)$  = colatitude in radians,  $\theta$

$Y(3)$  = radial component of index of refraction  
vector,  $\rho_r$

$Y(4)$  = latitude component of index of refraction  
vector,  $\rho_\theta$

$Y(5)$  = total path length in km,  $s$

$Y(6)$  = total path time delay in seconds,  $t$ .

For the calculation of the derivatives, as given by the ray tracing equations starting at Statement 100, these  $Y(I)$  are equivalent to  $R$ ,  $TH$ ,  $RHOR$ ,  $RHOTH$ ,  $S$ , and  $T$  respectively.

4.2.2 Data to Initialize Problem. Statements 1 through 4 are used to read in the sets of data to initialize the ray paths and to specify the ionospheric

model:

- M1 = number of data sets with different initial altitude, initial latitude, or model
- M3 = number of points to be computed for a given ray path
- M4 = number of different frequencies for specified initial altitude, latitude, and model
- M5 = number of different initial wave normal angles for specified initial altitude, latitude and model
- MODEL = integer specifying particular ionosphere model given in Statements 5 through 10
- R1 = initial altitude in km
- THH = initial latitude in degrees
- DELTA = number proportional to integration step size DELT; DELT varies as the inverse square root of the frequency

FREQ1(J) = wave frequencies in cps for which  
ray paths are to be calculated

DEL2(J) = wave normal angles in degrees with  
respect to the radius vector that  
initialize a ray path for each  
frequency. From the DEL2(J) the  
initial values of RHOR and RHOTH  
are computed.

4.2.3 Parameters that Specify Ionospheric Model. In order to compute the electron and ion plasma frequencies, a model for the variation of the ion and electron concentrations must be specified. The diffusive equilibrium model is given by equations (32) through (36) in Section 2.3. For the model specified by MODEL, the statements 5 to 10 parameterize the model:

RO = reference altitude from center of  
earth in km,  $r_0$

ANEO = electron density at RO in  $\text{cm}^{-3}$

ALFO(I) = Fraction concentrations of  $\text{H}^+$ ,  $\text{He}^+$ ,  $\text{O}^+$   
respectively at RO

TEMP       = ion temperature at RO in  $^{\circ}\text{K}$   
 GO          = acceleration of gravity at RO in  
             cm sec $^{-2}$   
 H(I)       =  $\text{H}^+$ ,  $\text{He}^+$ ,  $\text{O}^+$  scale heights in km.

4.2.4 Writeout of Initial Data. Statement 11 is the format for writing out the initializing quantities. Statement 12 writes the headings for the data at each point on the ray path.

4.2.5 Runge-Kutta (RK) Integration Routine. With Statement 23 the Runge-Kutta integration routine is begun. In the first transfer to Statement 100, the initial values of Y(1), Y(2) and DEL1 are used to compute RHOR (Y(3)), RHOTH (Y(4)), and the derivatives of the six ray tracing quantities D1(L). The statements KS = K and K = KS store the current value of K while the transfer is made out of a 'DO loop' then reassigns the value KS to K when the program is transferred back from 100. From these values of Y(L) and D1(L) the first predictor for each ray tracing quantity is computed PPP(L). In a like manner two more predictors QQQ(L) and VVV(L) and a corrector

SSS(L) are calculated to obtain the next point  $X(L, K + 1)$ . The quantity  $X(L, K)$  is used to store the current values of the ray tracing quantities. When  $K = 4$  the starting points and derivatives required for the Adams-Bashforth (AB) routine have been calculated and transfer is made to this routine. For  $K = 1, 2, 3$  each point on the ray path is written out as it is calculated. The integer MARK is equal to 1 after the integration step size has been reduced. The RK routine is then being used to start the AB routine with a new DELT. For MARK = 1 the value of KK is preserved and control is returned to Statement 27 with none of the points written out from the RK routine.

4.2.6 Adams-Bashforth (AB) Integration Routine. The four derivatives calculated by the RK routine are used to obtain the AB predicted values for the next point in Statements 27 and 28,  $P1(L)$ . These predicted are then used to compute new values of the derivatives from Statements 100 to 125.  $P1(3)$  and  $P1(4)$  are updated with the values of RHOR and RHOTH resulting from the  $P(1)$  and  $P(2)$ . In a

similar process the corrected values for the next point are calculated,  $C1(L)$ ,  $C1(3)$  and  $C1(4)$  are also updated by the new values of  $RHOR$  and  $RHOTH$ . To make sure none of the quantities  $Y(L)$  are changing too fast, the predictor and corrector are compared for each quantity in Statement 38. If the agreement is better than 0.01% the predictor and corrector values are used in Statement 32 to obtain the next point on the ray path. This point is written out in Statement 34. For a disagreement of greater than 0.01%  $DELT$  is halved and the RK routine is used to calculate the next three points with the reduced integration step size.

#### 4.2.7 Store Previous Value and Derivatives.

With the AB routine the derivatives necessary for computing the next point are stored from previous calculations. In Statement 37 the values of the derivatives are updated as well as the value of  $X(L,4)$ . In going to Statement 1000 the program is reinitialized to compute a ray path for the next initial wave normal angle or next frequency or next initial altitude and latitude.

4.2.8 Ray Tracing Equations. It is in this section from Statement 100 to Statement 125 that the expressions given in Section 2.3 are calculated. In order to save compilation and computation time this section of the program was not put into a subroutine. On an IBM 7044 error statements result because of the consequent transfer in and out of DO loops, but the program runs correctly. Each group of expressions can be identified with particular equations given in Section 2.3 as follows:

OM(I)	=	ion and electron gyrofrequencies in radians $\text{sec}^{-1}$ , equations (18) and (38)
DOMDR(I) and DOMDTH(I)	=	derivative of gyrofrequencies with respect to $r$ and $\theta$ , equations (39)
PLFREQ(I)	=	ion and electron plasma frequencies squared, calculated from equations (32) - (36) and equation (17)
ALHR	=	lower hybrid resonance frequency in cps given by

$$2\pi f_{\text{LHR}} = \left[ \frac{4\pi n_e e^2}{m_{\text{prot}}} \right]^{1/2} \Omega_e \left[ \alpha_1 + \frac{\alpha_2}{4} + \frac{\alpha_3}{16} \right] / [\pi_e^2 + \Omega_e^2]^{1/2} \quad (48)$$

and  $\alpha_i$  are the ion fractional concentration.

- DEL = angle between the radius vector and the wave normal; initially (K=1) DEL is specified by DEL1 and is used to obtain the initial values of RHOR and RHOTH
- PHI = angle between the radius vector and the magnetic field vector determined from BRO and BTHO the r and  $\theta$  components of the magnetic field;  $\gamma$  is equivalent to PHI in equation (41) and (43).
- PSI = angle between the magnetic field vector and the wave normal direction
- E(I) = sign of electronic charge for each specie, equation (19)
- AR, AL, P, SS, D, A, B, C, F = the quantities which are used to evaluate the phase index of refraction corresponding to equations (12), (13), (14), (16), (15), (8), (9), (10) and (11) respectively

AN2 = phase index of refraction square; the particular solution of equation (6) is chosen so that the error is minimized for B and F large; equation (7) is used for B < 0, the  $\pm$  sign determines the branch, the - generally gives the electron whistler branch.

RHOR,  
RHOTH = the r and  $\theta$  components of the index of refraction vector determined from AN1 and DEL and corrected to assure that  $\rho r^2 + \rho \theta^2 = n^2$

DADPSI, DBDPSI, DNDPSI = equations (23) and (22)

DNDROR,  
DNDROT = equations (21) and (20); for the case that  $\text{PSI} \rightarrow 0$  ( $< 10^{-8}$  numerically), DNDROR and DNDROT  $\rightarrow 0$

DRDPL(I), DLDPL(I), DPDPL(I), DADPL(I), DBDPL(I),  
DCDPL(I), and DNDPL(I) = equations (27), (26) and (25) respectively

DRDOM(I), DLDOM(I), DADOM(I), DBDOM(I), DCDOM(I),  
DNDOM(I) = equations (30), (29) and (28) respectively  
DPLDR(I) = equations (31) and (37)

DPSIDT = equations (40), (42) and (44); for the case that  $\psi \rightarrow 0$  ( $< 10^{-8}$  numerically) the form of DPSIDT is simplified

DNDR,  
DNDTH = equations (24)

DSDW, DPDW, DADW, DBDW, DCDW, DNDW = equations (47), (46) and (45) respectively

Dl(I) = derivatives of ray tracing quantities, equations (3), (4) and (5) respectively.

4.2.9 Data Input Cards to Initialize the Problem. The location of the input data numbers is specified with these comment cards. One example data set is listed under \$ENTRY SUI.

### 4.3 Performance of Program

The basic ray tracing computer program as listed in Section 4.1 calculates approximately 9 points of the ray path per second on an IBM 7044 computer.

If the ray is reversed by  $180^\circ$  it must retrace the same ray path. This feature of ray paths provides the means for testing the integration accuracy of the program in the case of the model ionosphere. Starting at  $30^\circ$ , 300 km, and  $\Delta_0 = 0$ , a ray path was calculated and then reversed at 300 km in the other hemisphere. After 1330 integration steps and a path length of 44,000 km, the ray returned to within  $0.2^\circ$  (0.6%) of its starting latitude and within  $0.06^\circ$  (0.03%) of its initial wave normal angle at 300 km. The one way integration error would be one half these values which is quite acceptable error for quantitative evaluation.

There is one situation which recurs in ray tracing which this program does not perform satisfactorily. Near a resonance at which the index of refraction goes to infinity or near a cut off at which it goes to zero,

the finite integration step size causes problems. If the ray path steps through the resonance or cutoff, the index of refraction squared  $AN^2$  becomes negative. Therefore  $n(AN1)$  and  $n^3(AN3)$  can not be computed and in fact the wave cannot propagate. Between Statements 106 and 110, 'Reduce Integration Step of Close to Resonance' in Section 4.1, the program is designed to test for this condition. If  $AN2$  is less than zero then the integration step  $DELTA$  is reduced by  $1/4$  and the Runge Kutta routine is used to recalculate the next point with the smaller integration step. With this method calculations can continue to closer approach the resonance or cutoff. For some cases even  $DELTA = 1 \times 10^{-8}$  is not sufficiently small to keep from having  $AN2$  go negative. Also for some cases the region for negative  $AN2$  is sufficiently narrow that the finite step size causes a point to be calculated on the other side of the resonance or cutoff where  $AN2$  is again positive. This situation is easily recognized since the ray tracing quantities are discontinuous.

#### 4.4 Example of Computer Printout and of a Ray Path

In Figure 2 is shown an example of the computer printout from the program listed in Section 4.1. At the top of the page are listed the initial data as read in for data set 1:

R1	=	initial altitude in km
TH1	=	initial latitude in degrees
DEL1	=	initial wave normal angle with respect to radius vector in degrees
FREQ	=	wave frequency in cps
M	=	maximum number of points to be calculated
DELTA	=	relative integration step size
RO	=	reference altitude for diffusive equilibrium ionospheric model in km
NEO	=	electron density in $\text{cm}^{-3}$ at RO
ALFO(1) and ALFO(2)	=	fraction concentration of $\text{H}^+$ and $\text{He}^+$ at RO
TEMP	=	ion temperature at RO in $^{\circ}\text{K}$ .

From these initial data the ray path trajectory of interest are calculated:

DATA SET = I		THI = 30.00DEG		DEL = 0.00DEG		FREQ = 200000.CPS		M =		500		DELTA = 10000.00	
R1 = 300.00KM		NEO = 2.000E 05CM-3		ALFO(1) = 0.00250		ALFO(2) = 0.02250		FE		TEMP = 800.00DEG K		DELT	
R = 400.00KM		DEL N		T		PSI		NE		FLHR		KK	
338.06000	0.00000	16.89394	0.00161	-41.11507	382839.18	987336.89	707.11	1	5774.67	707.11	1		
29.56544	0.53595	13.40993	0.00324	-40.85715	233823.79	963921.14	707.11	2	5709.34	707.11	2		
29.24961	1.42124	10.05463	0.00489	-40.33677	126396.30	934735.88	707.11	3	5785.55	707.11	3		
28.80607	3.03183	6.99612	0.00656	-39.24741	79991.21	896545.38	707.11	4	6472.54	707.11	4		
28.14158	5.88229	4.74235	0.00828	-37.18713	24523.56	844787.79	707.11	5	8576.42	707.11	5		
27.19288	8.98709	3.80145	0.01009	-35.23453	14007.63	780787.87	707.11	6	10059.89	707.11	6		
26.06574	11.32721	3.48543	0.01196	-34.30102	10282.29	716248.48	707.11	7	10195.45	707.11	7		
24.82240	13.86867	3.32333	0.01392	-33.36007	8165.49	656262.50	707.11	8	10045.58	707.11	8		
23.45724	15.98863	3.26990	0.01600	-33.05845	6831.39	601894.18	707.11	9	9761.45	707.11	9		
22.00645	17.91489	3.29233	0.01820	-33.13599	5955.19	554201.59	707.11	10	9390.34	707.11	10		
20.47925	19.67162	3.27339	0.02055	-33.57070	5356.64	513170.27	707.11	11	8986.35	707.11	11		
18.87723	21.30751	3.51149	0.02310	-34.32560	4934.19	478987.61	707.11	12	8593.01	707.11	12		
17.19654	22.87156	3.69985	0.02589	-35.37212	4632.72	449797.61	707.11	13	8237.76	707.11	13		
15.42683	24.40580	3.94587	0.02899	-36.69990	4422.92	426736.35	707.11	14	7936.72	707.11	14		
13.55084	25.94684	4.26118	0.03248	-38.31745	4290.41	409149.56	707.11	15	7700.48	707.11	15		
11.54293	27.52978	4.66513	0.03650	-40.25188	4231.09	397132.94	707.11	16	7538.58	707.11	16		
9.36643	29.19220	5.18823	0.04121	-42.55035	4250.46	391182.63	707.11	17	7463.13	707.11	17		
7.69619	30.97830	5.87893	0.04685	-45.28362	4367.19	392445.96	707.11	18	7492.21	707.11	18		
4.27752	32.94186	6.81549	0.05376	-48.50223	4624.82	403213.08	707.11	19	7653.04	707.11	19		
1.19218	35.14117	8.12910	0.06245	-52.47546	5127.84	427879.67	707.11	20	7978.90	707.11	20		
-2.38661	37.59125	10.03339	0.07342	-57.17369	6171.39	474397.49	707.11	21	8462.78	707.11	21		
-6.39406	40.12247	12.74813	0.08653	-62.51021	8741.92	552967.36	707.11	22	8875.60	707.11	22		
-10.19375	42.55362	15.44590	0.09939	-67.22854	16674.24	658196.50	707.11	23	8406.48	707.11	23		
-12.65612	46.00342	20.29020	0.10741	-68.18189	68717.50	742277.80	707.11	24	5227.60	707.11	24		
-14.32885	53.15039	17.11195	0.11036	-68.18189	107739.50	763672.66	707.11	25	4895.40	707.11	25		
-14.12234	61.43768	17.09199	0.11319	-55.27358	176899.09	786057.98	707.11	26	4769.03	707.11	26		
-14.60384	71.96806	15.86052	0.11561	-45.55613	221840.40	797810.79	707.11	27	4772.57	707.11	27		
-15.03991	83.99030	15.27972	0.11691	-34.26291	269392.72	808244.77	707.11	28	4794.23	707.11	28		
-15.43356	96.65662	15.35315	0.11895	-22.24974	343317.98	818152.88	707.11	29	4825.09	707.11	29		
-15.79221	108.82034	16.14589	0.12013	-10.67429	395582.71	828329.67	707.11	30	4864.08	707.11	30		
-16.11112	119.58696	17.58829	0.12169	-0.42790	448646.08	838705.44	707.11	31	4909.89	707.11	31		
-16.39070	128.59234	19.55984	0.12324	8.12495	606373.02	848988.71	707.11	32	4960.08	707.11	32		

Figure 2

# RAY PATH FOR 200 KHZ

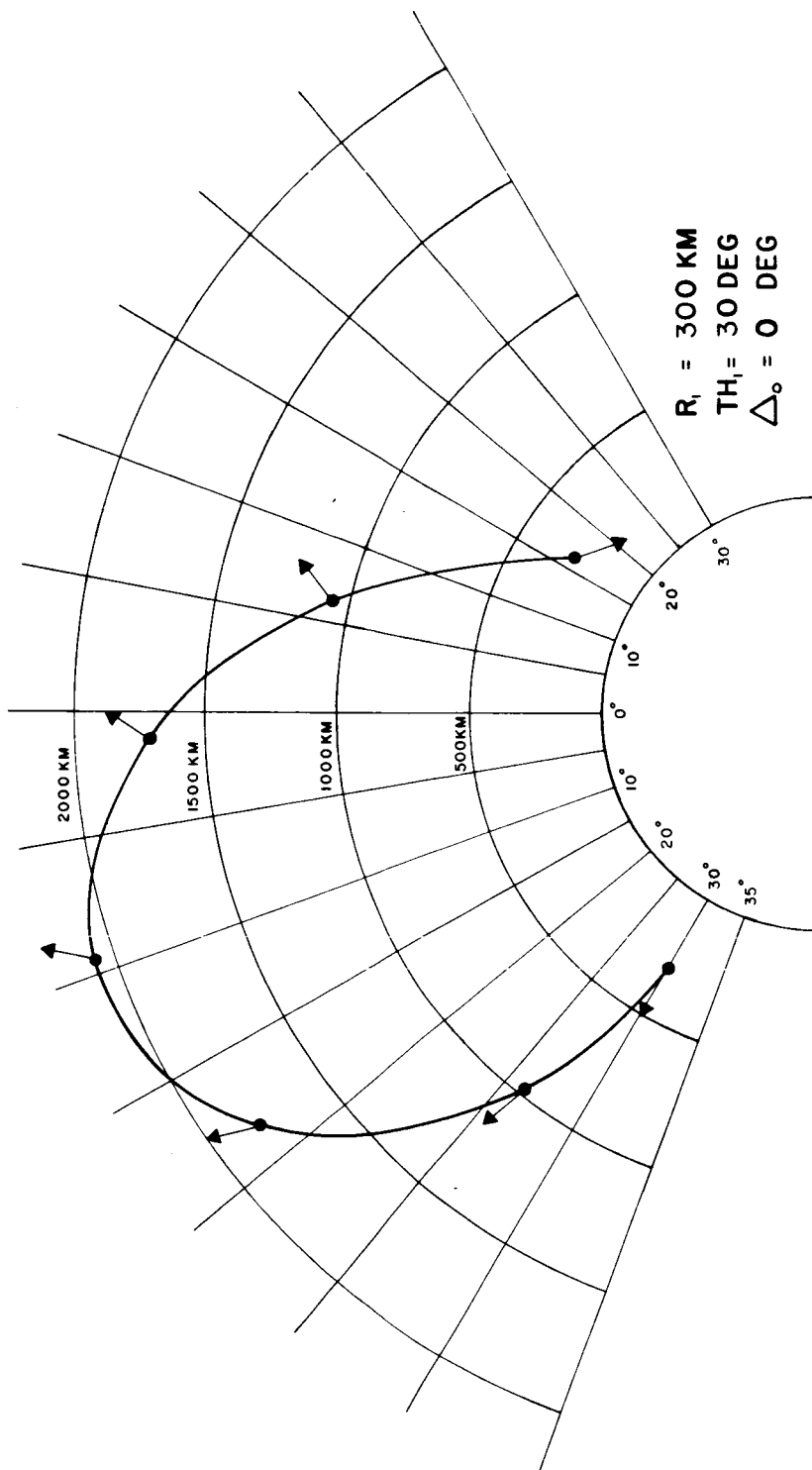


Figure 3

R	= altitude of ray path in km
TH	= latitude of ray path in degrees
DEL	= wave normal angle with respect to radius in degrees
N	= phase index of refraction
T	= group time delay to that point in seconds
PSI	= wave normal angle with respect to magnetic field
NE	= electron density in $\text{cm}^{-3}$
FE	= electron gyrofrequency in cps
FLHR	= lower hybrid resonance frequency in cps
DELT	= integration step size
KK	= number of point along ray path

For special ray tracing problems other computed quantities may be of interest.

In Figure 3 is shown a plot of the ray path listed in Figure 2. This plot is made in the polar coordinates  $r, \theta$  ( $R, TH$ ) with the direction of the wave normal indicated by the arrow at selected points.

From this plot several characteristics of ray paths can be noted:

- a. the wave normal and direction of the ray path

are not generally coincident

- b. the ray path does not follow the magnetic field line
- c. the ray path is not symmetric about the equator.

Plots of ray paths similar to that in Figure 3 seem to provide the best means of understanding the behavior of ray paths in a model ionosphere.

## 5.0 ADAPTATION OF PROGRAM FOR SPECIAL PROBLEMS

In Section 4.1 a basic ray tracing program for an idealized model ionosphere is given. For particular ray tracing programs such as those suggested in the Introduction (Section 1.0), the basic program can be modified. These modifications might include a routine to calculate all the ray paths between two specified points, a more realistic concentration model which includes latitude effects and possibly the 'knee' in the electron density, a distorted dipole magnetic field model, and a subroutine to do computer plotting of the resultant ray paths.

One of these modifications has been made to this basic program. The program has been used to find one or more ray paths between two specified points. This point-to-point routine can produce frequency-time spectrograms as a specified point ( $r_s, \theta_s$ ) for a source (lightning or VLF emission) at another specified point. For a given frequency, the parameter which is incremented is the initial wave normal direction to the radius vector  $\Delta_0$ .

- a. For each point on the ray path started with

an initial wave normal of  $\Delta_1$ , the distance between the satellite position  $(r_s, \theta_s)$  and that point is calculated,  $x_1$ .

- b. The magnitude of  $x_1$  is tested for each new new point and the ray path is stopped when  $x_1$  is a minimum  $XMIN_1$  and is positive if radially above the satellite and negative if below.
- c. For the next ray path  $\Delta_1$  is incremented by  $\Delta\Delta$  and  $XMIN_2$  calculated.
- d. The signs of  $XMIN_1$  and  $XMIN_2$  are tested; if the same, then both paths are either above or below the satellite. Then  $\Delta_2 \rightarrow \Delta_1$ ,  $XMIN_2 \rightarrow XMIN_1$  and  $\Delta_1$  is again incremented by  $\Delta\Delta$  until the signs are different. This condition implied that the two ray paths bracket the satellite.
- e. For this condition a new  $\Delta$  is obtained by a linear approximation from  $XMIN_1$  and  $XMIN_2$ :
 
$$\Delta_3 = \Delta_2 + XMIN_2 (\Delta_1 - \Delta_2) / (XMIN_1 - XMIN_2).$$
- f. When  $XMIN_3$  is less than 15 km from the satellite the ray path from the starting point to the

satellite point has been determined for that frequency. Values of  $\Delta_3$ ,  $XMIN_3$ , the frequency and the time delay are then printed out.

With this same routine ray paths could be determined between the ground and a specified point by incrementing the initial latitude for a given wave normal angle.

## 6.0 CONCLUSION

Because of the dipole magnetic field and the concentration gradients in the ionosphere, a model of the ionosphere for VLF ray tracing sufficiently complicates the ray tracing problems that ray paths must be calculated by numerical means. The Haselgrove ray tracing equations provide a set of equations which can be solved on a high speed digital computer.

In Section 2.0 the ray tracing equations are given along with the expressions necessary to evaluate for a model ionosphere with a dipole magnetic field and an  $H^+$ ,  $He^+$ ,  $O^+$ ,  $e^-$  diffusive equilibrium concentration model. The general flow of a ray tracing computer program is discussed in Section 3.0. A basic computer program for calculations in a meridian plane is listed and explained in Section 4.0. In Section 5.0 the addition of a routine to do point-to-point ray tracing calculations is discussed.

As cited in the Introduction, the numerical ray tracing approach has been used successfully to understand and to explain naturally occurring whistler phenomena. It is hoped that this basic ray tracing program can help

to further extend the knowledge about whistlers, VLF emissions, and VLF ground station transmissions as observed in the ionosphere.

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## DOCUMENT CONTROL DATA - R&amp;D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of Iowa Department of Physics and Astronomy		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	
		2b. GROUP	
3. REPORT TITLE A Computer Program for VLF Ray Tracing in a Model Ionosphere			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Progress May 1967			
5. AUTHOR(S) (Last name, first name, initial) Shawhan, Stanley D.			
6. REPORT DATE May 1967		7a. TOTAL NO. OF PAGES 72	7b. NO. OF REFS 23
8a. CONTRACT OR GRANT NO. Nonr-1509(06)		9a. ORIGINATOR'S REPORT NUMBER(S) U. of Iowa 67-12	
b. PROJECT NO.			
c.		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.			
10. AVAILABILITY/LIMITATION NOTICES Distribution of this document is unlimited			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Office of Naval Research	
13. ABSTRACT <p>The ray tracing equations and expressions necessary to evaluate these equations for a model ionosphere with a centered dipole magnetic field and an <math>H^+</math>, <math>He^+</math>, <math>O^+</math>, <math>e^-</math> diffusive equilibrium concentration model are given and discussed.</p> <p>A basic computer program for the numerical calculation of ray paths in a meridian plane is listed and explained. An additional routine to calculate point-to-point ray paths is briefly described. Examples are given of the computer printout for this ray tracing program and of a plotted ray path.</p>			

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14. KEY WORDS	LINK A		LINK B		LINK C	
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Computer Program						
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